Developing a Trigonometry Phone App

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

In this activity, students will be asked to develop a program for a smartphone application that could be used to calculate the missing side lengths and angle measures of a right triangle given specific sides or an angle. Students will discover multiple possibilities and then test their program. They must also create a PowerPoint presentation representing how the application will work. At the end of the task, students will present their projects to the rest of the class.

II. UNIT AUTHOR:

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III. COURSE:

Geometry

IV. CONTENT STRAND:

Triangles

V. OBJECTIVES:

The student will be able to:

- Solve problems involving right triangles, using sine, cosine, and tangent ratios.
- Solve practical problems, using right triangle trigonometry and properties of right triangles.

VI. REFERENCE/RESOURCE MATERIALS:

Ruler, Protractor, Calculator, Computer

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The assessment list for this activity is intended to evaluate the students' understanding of trigonometric equations. It will also evaluate the students' group work and final class presentation.

VIII. EVALUATION CRITERIA:

Assessment List for Activity 1, corresponding rubric.

IX. INSTRUCTIONAL TIME:

This Activity is estimated to take 2-90 minute blocks or 4-45 minute classes.

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Strand Trigonometry
Triangles

Mathematical Objective(s)
This activity will address the concept of trigonometry and right triangles. Students will develop a method for finding the missing side or angle of a right triangle based on given input. Students must develop a process for determining the correct trigonometric equation or Pythagorean Theorem. Students will also develop skills necessary to identify and evaluate different trigonometric equations. Students will also develop skills necessary to represent and analyze practical situations mathematically and graphically.

Related SOL

- G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will
  c) determine whether a triangle exists; and
  d) determine the range for the length of the third side.
- G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM Standards

- Use trigonometric relationships to determine lengths and angle measures.
- Draw reasonable conclusions about a situation being modeled.

Materials/Resources

- Ruler
- Protractor
- Calculator
- Computer
- Background/Instructions Page

Assumption of Prior Knowledge

- Students should have basic understanding of the following:
  - Pythagorean Theorem,
  - Trigonometric ratios and their applications in right triangles
  - Solving right triangle problems.
  - How to use PowerPoint.
- The relevant context students should have already explored is basic phone applications and process.

Potential Difficulties

- Students may have difficulty with and may need additional support with the following aspects of the project:
  - developing a basic program for filtering inputted information
  - determining which theorem or ratio to use
  - creating a PowerPoint presentation
  - speaking in front of the class

Introduction: Setting Up the Mathematical Task

- In the task, you will develop a basic smartphone application that will allow a person to calculate all of the remaining side lengths and angle measures of a right triangle when given specific input. To begin, you will be given an instructional page that sets the background for this task, which will discuss programming basics. In order to develop your program, you must create a process for filtering which ratio or theorem must be used in order to solve the problem. Also, you must be able to write trigonometric equations based on the input information. In your groups, you will discuss the necessary actions that need to be taken, such as determining what information is needed in order to use sine, cosine, or tangent. After gathering all the necessary information, your group will create a PowerPoint presentation describing your application and the developmental process. At the end of the task, your group will present to the class and discuss how your group came to that conclusion. You will be given roughly 180 minutes (two 90-minute blocks or four 45-minute periods) to gather information and create a presentation.
- Teacher will invite students to draw upon prior knowledge of phone application development by allowing a brief discussion of the topic. Also, a warm-up activity will be completed to help students understand the Pythagorean Theorem and trigonometric ratios.
- Teacher will guide students during the task by asking questions such as "What is this specific triangle?" "What information do you need in order to write a trigonometric equation?" "What is the formula to find a missing side of a right triangle given two sides?"
- Students will be working in groups. Within their groups, they will brainstorm different ideas to develop a process for solving right triangle problems. They will use cooperative learning and assign duties to each person in the group.
- Solving right triangle problems, by writing trigonometric equations and using the Pythagorean Theorem, and representing the findings visually help students develop necessary skills to be successful on the Geometry SOL.
- Presenting the projects of their group's findings will allow students to make their mathematical thinking and understanding public.

Student Exploration

Individual Work

- Students will complete a Warm-Up activity individually that activates prior knowledge of writing and solving right triangle problems involving trigonometric ratios and the Pythagorean Theorem.
- Students will also be assigned different duties, such as manager, coach, bookkeeper, editor, and referee, within groups that they are responsible for individually.

Small Group Work

- Students will work in small groups to accomplish the task of solving real-world situations involving right triangles by developing a smartphone application.
- Students will gather information needed to write trigonometric equations.
- Students will gather information needed to distinguish which process, trigonometric ratio or Pythagorean Theorem, needs to be used.
- Students will then test the program by solving the trigonometric equations based on the given input.
- Finally, students will create a PowerPoint presentation to represent the situation and findings.

Whole Class Sharing/Discussion

- Students will present their projects and findings to the entire class.
- Students will question each other's finding to gain a better understanding of different possibilities for finding a solution.

Student/Teacher Actions:

- Students will be working collaboratively in groups to develop an application to solve real-world right triangle problems.
- The teacher should be monitoring progress and ensure students are working collaboratively by checking with each group to see that each student has taken on one of the roles described below (manager, coach, bookkeeper, editor, and referee). In addition, the teacher should ask questions to confirm students' understanding. Below are sample questions.
    - How will your program find the third side length of a right triangle given two other side lengths?
    - How much information must your program be given to find all of the remaining angle measures and side lengths of a right triangle?
    - How is your group dividing the work between each member?
- One common misconception the students may experience is the confusion of using Pythagorean Theorem or trigonometric ratios when given a right triangle. They might not understand why one must use the Pythagorean Theorem when given two sides and asked to find the missing side. They also might not know which trigonometric ratio to use when given certain information.
- In order to address the misconception, the teacher should explain the consequences of switching the ratios and theorem.
- One way to increase student learning is to incorporate the use of computers for research on the topic. Students can use the computers to research trigonometric functions, the Pythagorean Theorem, solving right triangles, and also phone applications. They will also use the computers to create their projects.
- The use of graphing calculators should be promoted to help students verify algebraic results in their computations.

Monitoring Student Responses

- Students are to communicate their thinking and their new knowledge by creating and presenting PowerPoint projects to the class
- Students are to communicate with each other by working cooperatively in groups.
- Teacher and/or students are to highlight and clarify the ideas being grappled by answering questions asked by students.
- Teacher is to assist students who have difficulties by providing guidance and further explanations.
- Teacher is to extend the material for students that are ready to move forward and emphasize real-world application. One extension could possibly be allowing students to research application development tools and allow them to create an actual working app that could be downloaded from an app store and used on a phone. In addition, students could create a similar version of a working app in Microsoft Excel or Google Sheets by creating formulas in a spreadsheet that update as values are entered into it.

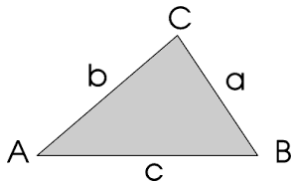
Summarize of the task/activity

- At the end of the task, students will be asked to present their findings to the class. Once all groups have presented, students will be asked to summarize all findings on an exit slip.
- In order to collect evidence of students' knowledge of the content described, the group projects created by the students will be posted in a digital and/or physical environment for other students to see. These projects will display the developmental process of the application, filtering aspect, writing of trigonometric or Pythagorean Theorem equations, and solving the equations.

Assessment List and Benchmarks

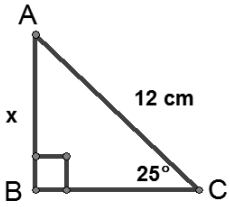
Warm-up Activity:

1. Given a=8 cm, b= 15 cm, and $\angle C = 90°$, find the length of c in cm: 2pts.

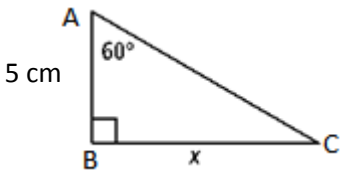


2. In the above figure if b = 5 cm and c = 13 cm, find the length of a in inches: 2pts.

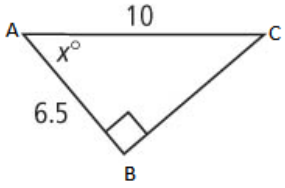
3. Given the side length of AC and measure of angle C, find side length AB in the picture below: 2pts.



4. Given the side length of AB. and measure of angle A, find side length BC in the picture below: 2pts.



5. Given the side length of AB and AC in cm, find the measure of angle A in the picture below: 2pts.



Task 1: Developing Trigonometric and Pythagorean Theorem Equations

In your groups, each person will be assigned a job. There are listed below:

- **Manager**- oversees the entire project to ensure accuracy and also completes the rubric.
- **Coach**- ensures everyone agrees on an answer.
- **Bookkeeper**- responsible for writing what everyone has agreed on and also completes the final presentation.
- **Editor**- ensures there are no errors, grammatically, graphically, or mathematically.
- **Referee**- ensures everyone is working together peacefully.

Each person in the group must participate and speak in the final presentation. In order to gain full credit, all categories on the rubric must be completed with accuracy. Use the rubric to self-assess to ensure all categories are met with full potential. Highlight the appropriate score you think your group should earn. Be sure to grade yourselves with honesty! This will be a group grade so collaboration is highly encouraged!

At the end of the task, your group must create a PowerPoint presentation of the real-world situation. In addition, your group must present these findings to the class. You must describe how you developed your program. In addition, each person in the group must explain some part of the task. Be sure to discuss the process of filtering information to determine which trigonometric equation must be used. Compare and contrast the different ratios and Pythagorean Theorem.

Developing a Trigonometry Phone App

Directions: Your goal is to collaboratively develop the mathematical groundwork for a phone app that, when given two side lengths or one side length and one angle measure of a right triangle, outputs the remaining side lengths and/or angle measures for the right triangle. The app should be given a name and a description as if it were available in an app store. The app should also be able to produce descriptive error messages if given numbers that are not applicable to right triangle trigonometry.

Background for Developing an App

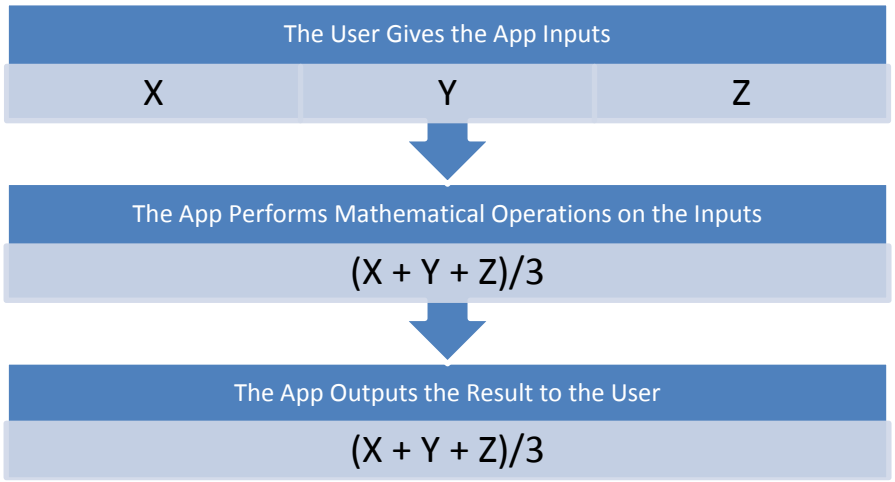
When creating your app, it is important to remember that the goal of the app is to calculate all missing sides and angles for a right triangle when the user inputs two side lengths, or one side length and one angle. Your group needs to make sure that your app knows what to do in any circumstance. Some guiding questions are below.

- How will the app calculate the remaining side and angles when the user inputs the length of the legs of the right triangle?
- How will the app calculate the remaining sides and angles when given a leg length and hypotenuse length of the right triangle?
- How will the app calculate the remaining sides and angles when given a side length and an angle measure of the right triangle?

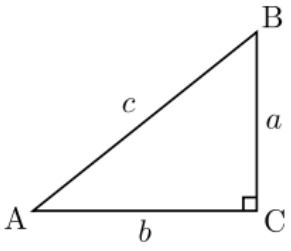
Note that because you do not know what the user's input will be, your app should use algebraic expressions to represent them.

For example, if we were creating an app to find the average of three numbers, we would need three **inputs** from the user. The user would type into the app any three numbers. We could call these numbers $x, y,$ and z.

Then we could simply tell the app to **output** $\frac{x+y+z}{3}$. The following flow chart represents how this app would work.



Below is an example of how an app might find the perimeter and area of a right triangle given the length of two sides of the right triangle. It is helpful to use the following picture as a reference.



If given sides a and b, then the area is $\frac{ab}{2}$ and the perimeter is $a + b + \sqrt{a^2 + b^2}$.

If given sides a and c, then the area is $\frac{a\sqrt{c^2-a^2}}{2}$ and the perimeter is $a + c + \sqrt{c^2 - a^2}$.

If given sides b and c, then the area is $\frac{b\sqrt{c^2-b^2}}{2}$ and the perimeter is $b + c + \sqrt{b^2 - a^2}$.

All in all, most of your work will involve deciding how your app will take in **inputs** about side lengths and angle measures of a right triangle**, perform mathematical operations** on those inputs, and **output** information to the user about the remaining side lengths and angle measures of the right triangle.

Rubric:

RUBRIC FOR TASK 1						
Group Members:					**Assessment Grade**	
#	**Element**	**0**	**1**	**2**	**Self**	**Teacher**
1	App title and description	No title or description provided	Title and description are incomplete	Title and description provided		
2	The group clearly communicates how the app takes in information and outputs results.	No explanation of how the app works	Unclear explanation of how the app works	Clear explanation of how the app works		
3	The app outputs all correct information when given two side lengths.	App does not output correct information	App outputs correct information for some cases	App outputs correct information for all cases		
4	The app outputs all correct information when given a side length and an angle measure.	App does not output correct information	App outputs correct information for some cases	App outputs correct information for all cases		
5	The app outputs descriptive error messages for impossible side lengths or angle measures.	App does not output error messages	App outputs error messages for some impossible cases	App outputs error messages for all impossible cases		
6	Neatness of materials	Materials are not neat	Materials are somewhat lacking in neatness	Materials are neat and easy to read.		
7	The components of the project are organized.	No organization	Some organization	Materials are well-organized when handed in.		
8	Group Duty	Did not complete duty	Completed duty but inaccurately	Completed duty accurately		
9	Speech	No Speech	Not everyone spoke clearly and understandably	Everyone spoke clearly and was understandable		
10	Presentation of Findings	No presentation	Presented findings but not accurately	Presented findings accurately		
TOTAL POINTS EARNED					/20	/20

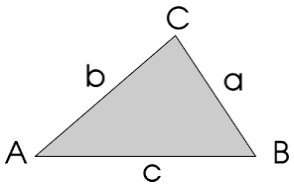
Exit Slip:

1. After reviewing every group's findings, was there a common process for writing equations?
2. Did any group have the same system of equations? If so, was their solution the same?
3. Summarize the general findings of each group's presentation.

Sample Student Work:

Warm-up Activity:

1. Given a=8 cm, b= 15 cm, and $\angle C = 90°$find the length of c in cm: 2pts.

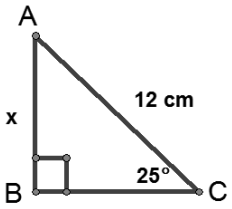


$$8^2 + 15^2 = c^2$$
$$64 + 225 = c^2$$
$$289 = c^2$$
$$\boxed{17\ cm = c}$$

2. In the above figure if b = 5 cm and c = 13 cm, find the length of A: 2pts.

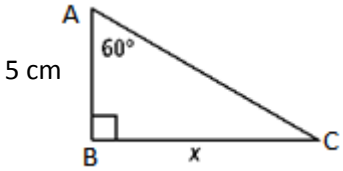
$$a^2 + 5^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 = 144$$
$$\boxed{a = 12\ cm}$$

3. Given the side length of AC and measure of angle C, find side length AB in the picture below: 2pts.



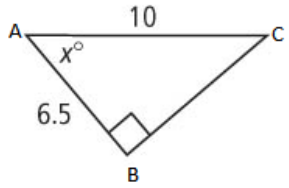
$$\sin(25°) = \frac{x}{12}$$
$$12 \cdot \sin(25°) = x$$
$$\boxed{5cm \approx x}$$

4. Given the side length of AB. and measure of angle A, find side length BC in the picture below: 2pts.



$$\tan(60°) = \frac{x}{5}$$
$$5 \cdot \tan(60°) = x$$
$$\boxed{8.7\ cm \approx x}$$

5. Given the side length of AB and AC in cm, find the measure of angle A in the picture below: 2pts.



$\cos(x°) = \frac{6.5}{10}$

$x° = \cos^{-1}\left(\frac{6.5}{10}\right)$

$\boxed{x° \approx 49.5°}$

Sample Student Work:

Group Members and Assignments:

Maggie – Manager

Carl – Coach

Bob – Bookkeeper

Erin – Editor

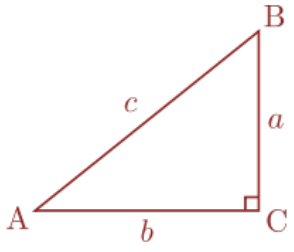
Rachel – Referee

App Title: Right Triangle Calculator

Description: This app allows the user to find all of the remaining side lengths and angle measures of a right triangle when given certain information about the right triangle.

How the app works:

The app works by first asking for either two side lengths or one side length and one angle measure in degrees. We thought it would work best by showing the user a picture like the one below, and letting the user type in whatever given information they have.



Type in your given information. (The rest will be filled in by the app.)

a = _____ $m\angle A =$ ____°

b = _____ $m\angle B =$ ____°

c = _____ $m\angle C = 90°$

Then the app would work on a case-by-case basis depending on what information the user enters. The cases are listed below. We chose to use the given information as often as possible.

Case 1: Given a, b, and $m\angle C = 90°$, then $c = \sqrt{a^2 + b^2}$, $m\angle A = \tan^{-1}\left(\frac{a}{b}\right)$, and $m\angle B = \tan^{-1}\left(\frac{b}{a}\right)$.

Case 2: Given a, c, and $m\angle C = 90°$, then $b = \sqrt{c^2 - a^2}$, $m\angle A = \sin^{-1}\left(\frac{a}{c}\right)$, and $m\angle B = \cos^{-1}\left(\frac{a}{c}\right)$.

Case 3: Given b, c, and $m\angle C = 90°$, then $a = \sqrt{c^2 - b^2}$, $m\angle A = \cos^{-1}\left(\frac{b}{c}\right)$, and $m\angle B = \sin^{-1}\left(\frac{b}{c}\right)$.

Case 4: Given A, a, and $m\angle C = 90°$, then $m\angle B = 90 - A$, $b = \frac{a}{\tan(A)}$, and $c = \frac{a}{\sin(A)}$.

Case 5: Given A, b, and $m\angle C = 90°$, then $\angle B = 90 - A$, $a = b \cdot \tan(A)$, and $c = \frac{b}{\cos(A)}$.

Case 6: Given A, c, and $m\angle C = 90°$, then $m\angle B = 90 - A$, $a = c \cdot \sin(A)$, and $b = c \cdot \cos(A)$

Case 7: Given B, a, and $m\angle C = 90°$, then $m\angle A = 90 - B$, $b = a \cdot \tan(B)$, and $c = \frac{a}{\cos(B)}$

Case 8: Given B, b, and $m\angle C = 90°$, then $m\angle A = 90 - B$, $a = \frac{b}{\tan(B)}$, and $c = \frac{b}{\sin(B)}$

Case 9: Given B, c, and $m\angle C = 90°$, then $m\angle A = 90 - B$, $a = c \cdot \cos(B)$, and $b = c \cdot \sin(B)$

Error Messages:

The app would give the following error messages.

Error 1: If given zero or a negative number for angles measures or side lengths, then the app will display "Side lengths and angle measures must be positive."

Error 2: If given two side lengths where a leg is longer than the hypotenuse, then the app will display "The length of c must be more than a and b."

Error 3: If given an angle measure that is greater than or equal to 90°, then the app will display "The value for A and B must be less than 90°."